## **UNIVERSITY OF MICHIGAN**

## **Department of Civil & Environmental Engineering**

# **Evaluation of the Efficiency of Shear Studs for Punching Shear Resistance of Slab-Column Connections**

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## **1. INTRODUCTION**

### 1.1 SLAB COLUMN FRAMED STRUCTURES

Reinforced-concrete framed structures in which slabs are supported directly on columns without the use of beams or girders, are referred to as slab-column framed structures (Fig. 1-1). The structures are efficient and economical, commonly used for apartments, office and institutional buildings (i.e. schools and hospitals) because of their low cost and reduced story heights.



(http://nisee.berkeley.edu/elibrary/Image/GoddenF80) Fig. 1-1 Slab Column Framed System

### **1.2 CHALLENGES IN THE DESIGN OF SLAB-COLUMN FRAMED SYSTEMS**

Unacceptable deflections and punching shear failures around columns are two main challenges facing the design of slab-column framed systems. Deflection-related problems in flat slabs are often solved by applying prestressing to the slab. However, punching shear failures around connections are still of concern in seismic region, where the slab-column frames will undergo gravity- and earthquake-induced shear stresses and lateral displacement. Because punching shear failures occur suddenly, they must be prevented.

### **1.3 SHEAR FAILURE MODES IN FLAT SLABS**

Two kinds of shear failure modes may be critical in flat slabs. The first is one-way shear failure or beam-action shear involves an inclined crack extending across the entire width of the slab, as shown schematically in Fig. 1-2a. Alternatively, failure may occur by punching shear, with the potential diagonal crack following the surface of a truncated cone or pyramid around the column, as shown schematically in Fig. 1-2b.



Fig. 1-2 Shear Failure Modes in Slabs

#### **1.4 SHEAR REINFORCEMENT IN SLAB-COLUMN CONNECTIONS**

#### 1.4.1 Bent-up bars

Generally, bent-up bars, as shown in Fig. 1-3, are 135° inclined bars that are used to resist tension in the bottom of the beam near mid-span. They may modestly improve the ultimate punching shear resistance of slab-column connections when they are well anchored (Hawkins, 1974; Ghali and Hammill, 1992; Polak, et al., 2005). However, bent-up bars do not seem to be effective in increasing connection deformation capacity (Broms, 2000).



#### 1.4.2 Closed Stirrups

ACI 318-08 R11.11.3 allows the use of closed stirrups, provided there are longitudinal bars in all four corners of the stirrups, as shown in Fig. 1-4. They have been found effective in increasing the shear strength capacity and deformation capacity of slab-column connections. However, installment is labor-intensive and expensive, particularly in slabs with thickness less than 10 in. (MacGregor, 2005).



#### **1.4.3 Structural Steel Shearheads**

Structural steel shearheads shown in Fig.1-5 consist of standard I shape embedded in the slab and projecting beyond the column. They are employed to increase the effective perimeter  $b_0$  of the critical section for shear, thus increasing the shear capacity of the connections. However, interference problems and installation difficulties are two main problems of shearheads reinforcement (Dilger and Ghali, 1981).



(adapted from Nilson et al., 2004) Fig. 1-5 Shearheads

#### 1.4.4 Shear Studs

Headed shear-stud reinforcement at a slab-column connection consists of rows of vertical rods, each with a circular head or plate welded, as shown in Fig. 1-6. These rows are placed extending out from the corners of the column. To facilitate handling and placement of the shear studs and to anchor the lower ends of the studs, they are generally shop-welded to flat steel bars at the desired spacing. The vertical rods are referred to as headed shear studs. Results reported in the previous tests (Elgabry and Ghali, 1987; Megally and Ghali, 2000; Robertson et al., 2002) suggested that shear studs should perform well.



(http://www.vsl.net/construction\_systems/shear\_rail.html) Fig. 1-6 Headed Shear Studs

## 2. RESEARCH MOTIVATION AND OBJECTIVES

#### **2.1 RESEARCH MOTIVATION**

Punching shear failure is a brittle failure mode in slab-column connections. Once a punchingshear failure has occurred at a slab-column joint, the shear capacity of that particular joint is almost completely lost and therefore, it must be prevented in regions of seismicity.

Several types of shear reinforcement (e.g. bent-up bars, closed stirrups, steel shearheads) have been evaluated for use to increase punching shear capacity in slab-column connections. However, installation of such reinforcement in relatively thin flate plates is difficult, and providing adequate anchorage is a problem. Currently, shear stud reinforcement is the most widely used shear reinforcement in the United States because of the easy installment in construction. Experiment results from previous test (Elgabry and Ghali, 1987; Megally and Ghali, 2000; Robertson et al., 2002) seem to confirm that shear stud reinforcement should perform well for increasing punching shear resistance of slab-column connections subjected to uni-axial displacement reversals. However, there's only limited information about the behavior of slabcolumn connections with shear stud reinforcement under bi-axial lateral displacements.

In 2009, a dissertation "Punching Shear Strength and Deformation Capacity of Fiber

Reinforced Concrete Slab-Column Connections under Earthquake-Type Loading," by Min-Yuan Cheng showed that the results from the test of Specimen SB3 with shear studs reinforcement under combined gravity load and bi-axial lateral displacements indicated that current provisions in the ACI Building Code 318-08 for the design of shear stud reinforcement in slab-column connections subjected to earthquake-induced displacements are potentially unsafe. Cheng found the use of  $V_c = 3\sqrt{f_c'}$  (psi) ( $V_c$  is the nominal shear strength provided by concrete and  $f_c'$  is the compressive strength of concrete) may not be conservative in calculating the punching shear capacity of the connection. Instead, the use of  $V_c = 2\sqrt{f_c'}$  (psi), as for connections with shear reinforcements other than shear studs would have led to a slightly conservative estimation of punching shear capacity in the test specimen with shear stud reinforcement. Moreover, the shear stud reinforcement seems to have little or no effectiveness in increasing connection ductility. Given these factors, further studies are needed to assess the efficiency of shear studs for punching shear resistance of slab-column connections.

#### **2.2 RESEARCH OBJECTIVES**

The main objective of the research was to evaluate the effectiveness of shear studs in improving punching strength and connection ductility when subjected to gravity- and earthquake-induced lateral displacements.

In the summer of 2010, two slab specimens with different reinforcement ratio of 0.8% and 1.2% were tested under monotonically increased concentrated load. The main objectives of this research stage were: 1) to evaluate the influence of flexural reinforcement ratio on punching shear strength of slab-column connections subjected to monotonically increased concentrated load. 2) to evaluate the effectiveness of shear studs for increasing punching shear strength of slab-column connections subjected to monotonically increased concentrated load.

## **3. EXPERIMENTAL PROGRAM**

#### **3.1 TEST SPECIMENS**

The two specimens, representing isolated interior slab-column connections, were tested under monotonically increased load. The applied load on the slab was intended to simulate, in a simple manner, shear and moment effects on the slab-column connections of a regular two-way flat plate system.

The slab dimensions were the same for the two specimens, 72\*72\*8 inches, with a 6 in. square column stub at the center of the slab for load application. The plan and elevation views of the specimens are shown in Fig.3-1 and Fig.3-2, respectively. A photo of one of the specimens is shown in Fig. 3-3.



Fig. 3-1 Plan View

Fig. 3-2 Elevation View



Fig. 3-3 Photo of Specimens

The reinforcing bars were different in the two specimens. For Specimen 1, the reinforcing bars were #4 bars (diameter = 0.5 in.) made of Grade 60 steel (nominal yield strength is of 60 ksi), and the reinforcement ratio was 0.78% (#4 bars at 4 in. spacing). For Specimen 2, the reinforcing bars were #5 bars (diameter= 0.625 in.), also made of Grade 60 steel, and the reinforcement ratio was 1.2% (#5 bars at 4 in. spacing). The effective depth d, taken as the average for both reinforcement directions, was 6.5 in. and 6.38 in. for Specimen 1 and Specimen 2, respectively.

The shear studs were the same for both specimens. A total of eight rails were used in each specimen. The first stud was at 2 in. from the column face, while subsequent studs were spaced at 4.75". The stud details are shown in Fig. 3-4.



Fig. 3-4 Stud Details

#### **3.2 MATERIAL PROPERTIES**

Ready-mix concrete was used in the two specimens. Average compressive strength of the concrete used in the slab specimens, measured from cylinder (4 x 8 in.) tests, were 5550 and 6200 psi for Specimen 1 and Specimen 2, respectively. Grade 60 reinforcing bars were used for slab reinforcement. Measured yield strengths for the slab flexural reinforcement were 63.7 and 60.8 ksi for Specimen 1 and Specimen 2, respectively. Also, measured yield strength for head studs was 77.9 ksi. Table 1 presents the material properties of the specimens.

Table	1Material	<b>Properties</b>
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Specimen	Concrete strength (psi)	Bar yield strength (ksi)	Stud yield strength (ksi)	Flexural reinforcement ratio (%)
1	5550	63.7	77.9	0.8
2	6200	60.8	77.9	1.2

#### **3.3 TEST SETUP**

A vertically oriented hydraulic Jack, connected to a steel reaction frame, was used to apply monotonically load to the column stub of each slab specimen, as shown in Fig. 3-5. The test specimens were supported along their perimeter on a 0.5 in. thick neoprene pad placed on top of a steel tube in order to simulate a simply supported boundary condition.



(Sources: Alex DaCosta, MI)

Fig. 3-5 Test Equipment: Actuator

Eight strain gauges were attached to the reinforcing bars of each specimen at locations adjacent to the column stub. The location of the strain gages is shown in Fig. 3-6. Four strain gauges (two in each principal direction) were located at 0.5d (d is the slab effective depth, i.e. the distance from extreme compression fiber to centroid of longitudinal tension reinforcement) away from the column faces. The remaining four strain gauges were located on the same reinforcing bars, but at 1.5d away from the column faces.



Fig. 3-6 Gauged Bar Locations

Twenty strain gauges were attached to the shear studs of each specimen at locations adjacent to the column stub. The location of the strain gauges is shown in Fig. 3-7.



Fig. 3-7 Gauged Studs Locations

## 4. EXPERIMENTAL DATA AND ANALYSIS

### 4.1 OBSERVED DAMAGE

The specimens are flipper over by crane at the end of the test. Crack patterns at failure for both specimens on bottom (tension) side are shown in Fig. 4-1. In both tests, the column stub was



(a) Specimen 1

(b) Specimen 2

Fig. 4-1: Crack Pattern on Slab Tension Side

clearly observed to punch through the slab. However, the observed failure pattern was not a punching cone as typically assumed.

### **4.2 THEORETICAL STRENGTH**

The theoretical punching shear capacity was calculated based on ACI Building Code 318-08. The theoretical punching shear capacity of Specimen 1 and Specimen 2 was 145 and 146 kips, respectively, indicating the same punching shear capacity. The detailed calculations can be found in the Appendix B.

The theoretical flexural strength was calculated by yield line analysis (Elstner and Hognestad, 1956). The theoretical flexural strength of Specimen 1 and Specimen 2 was 142 and 210 kips, respectively. The detailed calculations can be found in the Appendix C. Table 2 gives a comparison between theoretical punching shear strength and theoretical flexural strength for both specimens.

Specimen	Theoretical Punching	<b>Theoretical Flexural</b>	<b>Predicted Failure</b>
	Shear Strength (k)	Strength (k)	
1	145	142	Punching and Flexural
2	146	210	Punching

Table 2 – Comparison between theoretical punching shear strength and theoretical flexural strength

For Specimen 1, both punching shear failure and flexural failure were predicted, because the theoretical punching shear capacity and theoretical flexural strength were almost equal. For Specimen 2, punching shear failure and limited yielding of reinforcing steel bars were predicted. Because the theoretical flexural strength was much larger than the theoretical punching shear capacity, it is assumed herein that the slab has adequate flexural reinforcement to prevent flexural failure.

### 4.3 TEST LOAD AND DEFLECTIONS

Fig. 4-2 showed the applied load P versus deflection responses for the two slab specimens. It was observed that the peak load achieved during the testing was 127 kips and 151 kips for Specimen 1 and Specimen 2, respectively.

At the very beginning, the overlapping curve in Fig. 4-2 indicated that both the slab and column moved together. However, after the peak load, the separation of the curves indicated that punching had occurred.



(a) Specimen 1



(b) Specimen 2 Fig. 4-2 Load versus Deflection Response

Table 3 summarizes the load and deflection at peak for both specimens.

Specimen	Material Properties	Bar Spacing (in.)	Reinforce ment Ratio (%)	Concrete Strength (Psi)	Peak Load, P <sub>max</sub> (k)	Normalized Shear Stress $\frac{P_{max}}{b_0 d \sqrt{f'_c}}$	Deflection at Peak Load (in.)
1	Regular	4	0.8	5550	127	5.2	0.48
2	Concrete	6	1.2	6220	151	6.1	0.36

Table 3- Load and Deflection at Peak

The test results presented herein showed that larger flexural reinforcement ratio contributes more strength to the slab-column connections, which was not expected from the ACI Building Code 318-08. Because the ACI code considers only the flexural depth ( $d_{avg}$ ), which is larger in Specimen 1 than in Specimen 2 (because of smaller diameter flexural bars), and not explicitly account for the effect of flexural reinforcement on punching strength, it assigns Specimen 1 the same theoretical punching capacity as Specimen 2, which contradicts to what actually occurred in the tests. The fact is that immediately after inclined cracking, a certain portion of the total shear is carried by  $V_d$ , the dowel action of the longitudinal reinforcement, and  $V_{ay}$ , the vertical component of the shear transferred across the crack by interlock of the aggregate particles on the two faces of the crack. As crack widens,  $V_{ay}$  decreases, increasing the fraction of the shear resisted by  $V_d$ . Since larger flexural bars showed greater dowel action, it increases the shear strength of the slab-column connections to a certain extent.

Also, Specimen 2 (reinforcement ratio = 1.2%) showed greater initial stiffness compared with Specimen 1 (reinforcement ratio = 0.8%). However, because failure in Specimen 2 occurred at smaller deflection, the Specimen showed less ductility than Specimen 1.

For comparison purposes, Fig. 4-3 showed the specimen responses in terms of the average punching shear stress (*P*/*b*<sub>0</sub>*d*), normalized by the square root of the concrete cylinder strength,  $f'_c$ . The normalization is meant to reduce the effect of different concrete cylinder strength. The critical perimeter,  $b_o$ , was calculated according to the ACI Building Code 318-08 and was equal to 50 in. for Specimen 1 and 49.5 in. for Specimen 2. The normalized shear stress for both specimens is presented in Table 3. Since Specimen 2 displayed larger normalized shear stress (6.1) than Specimen 1 (5.2), this suggested that slab-column connection with higher reinforcement ratio would have larger normalized punching shear strength. Also, it should be notable that the normalized strength values of both specimens at peak are greater than  $4\sqrt{f'_c}$  (the punching shear strength of concrete in absence of shear reinforcement (ACI 318-08)). Since  $4\sqrt{f'_c}$  is generally conservative, it is hard to determine whether shear studs increased the punching shear strength of the connections or not. Control specimens (slab without flexural reinforcement) are needed to give further conclusions.



(a) Specimen 1



(b) Specimen 2 Fig.4-3 Normalized Stress versus Deflection Response

### 4.4 TEST LOAD AND STUD STRAIN

Strain in the shear studs were measured by 20 strain gauges located around the connection region (Fig. 3-7). Plots of load vs. stud strain at different rows for Specimen 1 are shown in Fig. 4-4.



(a) First Row



(b) Second Row



(c) Third Row

Fig. 4-4 Load vs. Stud Strain for Specimen 1

In the test, strains were negligible prior to flexural cracking in the slab. Beyond cracking and prior to yielding, they are basically proportional to the applied load. When the applied load was increased beyond 60 k, substantial decrease in stiffness and increase in strain were observed, indicating flexural cracks are initiated at the bottom (tension) surface of Specimen 1. When the applied load was increased to 120 k, punching shear failure occurred after inclined cracks extended from the bottom surface of the slab diagonally upward to the top surface.

Different behavior of studs was observed in Fig. 4-4. Studs as SER3 and SNR3 began to yield and deform a large amount before failure while some studs as SWL1 and SER2 showed limited deformation. The different behavior showed the sensitivity of stud strain to the location of cracks. Also, it seemed that only studs in the location of inclined cracks were effective in bridging the cracks. The remaining studs just serve to bridge the flexural cracks or even not work at all.

## **5. CONCLUSIONS**

The test results presented herein showed larger flexural reinforcement ratio contributed more strength to the slab-column connections with shear stud reinforcement, which was not expected from the ACI code. The tests also showed that specimen with larger flexural reinforcement ratio exhibits higher initial stiffness and less ductility before failure than its counterpart specimen with lower flexural reinforcement ratio.

Both specimens failed in brittle fashion. The observed failure pattern was not a punching cone as typically assumed. In most instances, the studs were not crossed by the critical inclined cracks, thus lowering their overall effectiveness to improve punching shear strength.

More specimens need investigating in future to give further conclusion of the effectiveness of shear studs in improving punching shear strength for slabs.

#### 6. APPENDICES Appendix A: ACI DESIGN EQUATIONS:

11.1.1- Design of cross sections subjected to shear shall be based on

$$\mathbf{\phi} \mathbf{V}_{\mathbf{n}} \ge \mathbf{V}_{\mathbf{u}} \tag{11-1}$$

where  $\Phi$ =0.75 for shear,  $V_u$  is the factored shear force at the section considered and  $V_n$  is nominal shear strength computed by

$$\mathbf{V}_{\mathbf{n}} = \mathbf{V}_{\mathbf{c}} + \mathbf{V}_{\mathbf{s}} \tag{11-2}$$

where  $V_c$  is nominal shear strength provided by concrete and  $V_s$  is nominal shear strength provided by shear reinforcement.

**11.9.9.1-** Where  $V_u$  exceeds  $\phi V_c$ , horizontal shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where  $V_s$  shall be computed by

$$\mathbf{V}_{\mathbf{s}} = \frac{A_v f_y d}{s} \tag{11-29}$$

Where  $A_v$  is area of horizontal shear reinforcement with spacing s, and d is determined in accordance with 11.9.4.

**11.11.5.1**- For the critical section defined in 11.11.1.2,  $V_n$  shall be computed using Eq. (11-2), with  $V_c$  not exceeding  $3\sqrt{f'_c}b_0d$  for connections with headed shear stud reinforcement.

#### **Appendix B: Punching Shear Capacity Calculations**

For Specimen 1:  $d_{avg} = h-1"-0.5" = 8"-1"-0.5" = 6.5"$   $b_0 = 4*(6"+6.5") = 50"$   $A_v = 8*A_b = 8*0.11 \text{ in}^2 = 0.88 \text{ in}^2$   $V_s = A_v*f_{yt}*\frac{d}{s} = 0.88 \text{ in}^2 * 60 \text{ ksi} * \frac{6.5"}{4.75"} = 72.3 \text{ kips}$   $V_c = 3\lambda\sqrt{f_c} b_0 d = 3*1*\sqrt{5550 \text{ psi}} * 50"* 6.5" = 72.6 \text{ kips}$  $V_n = V_s + V_c = 145 \text{ kips}$ 

For Specimen 2:  $d_{avg} = h-1"-0.625" = 8"-1"-0.625" = 6.38"$   $b_0 = 4*(6"+6.38") = 49.5"$   $A_v = 8*A_b = 8*0.11 \text{ in}^2 = 0.88 \text{ in}^2$   $V_s = A_v*f_{yt}*\frac{d}{s} = 0.88 \text{ in}^2 * 60 \text{ ksi} * \frac{6.38"}{4.75"} = 70.9 \text{ kips}$   $V_c = 3\lambda\sqrt{f_c} b_0 d = 3*1*\sqrt{6200 \text{ psi}} * 49.5"* 6.38" = 74.6 \text{ kips}$  $V_n = V_s + V_c = 146 \text{ kips}$ 

Note: The ACI code limits  $f_{yt} = 60$  ksi for calculations, while the actual  $f_{yt}$  was 77.9 ksi for the two specimens.

### Appendix C: Flexural Strength by Yield-Line Analysis

(Adapted from Elstner and Hognestad, 1956)

Yield-Line Pattern for test specimens subjected to direct increased concentrated load (The slab's corners are free to lift):



(Elevation View)

Assume deflection 1 at column stub

Internal Energy:  $4m(L-2x)\frac{2*1}{L-r} + 4mx\frac{2*1}{L-r-x}$ External Energy: P\*1

Set Internal Energy = External Energy P = 4m  $(L-2x)\frac{2*1}{L-r} + 4mx\frac{2*1}{L-r-x}$ 

Calculate x for minimum P:  $\frac{\partial P}{\partial x} = 0$ 

So x = 
$$(1-0.5*\sqrt{2})(L-r)$$
  
P = 8m  $(\frac{1}{1-\frac{r}{L}}-3+2\sqrt{2})$ 

For Specimen 1: Since  $m = d^{2*}p^*f_{yp}^*(1-0.59^*q) = (6.5)^{2*}0.00755^*63.7^*(1-0.59^*0.0874) = 19.3 \text{ k-ft/ft}$ 

(Note: 
$$p = \frac{A_s/ft}{bd} = \frac{3*0.2}{12*6.5} = 0.00755, q = \frac{p*f_{yp}}{f_c} = \frac{0.00755*63.7}{5.5} = 0.0874$$
)  
 $P = 8m(\frac{1}{1-\frac{r}{L}} - 3 + 2\sqrt{2}) = 8*19.3*(\frac{1}{1-\frac{6}{72}} - 3 + 2\sqrt{2}) = 142 \text{ k}$ 

For Specimen 2: Since  $m = d^{2*}p*f_{yp}*(1-0.59*q) = (6.38)^2*0.01215*63.7*(1-0.59*0.1343) = 29.0 \text{ k-ft/ft}$ 

(Note: 
$$p = \frac{A_s/ft}{bd} = \frac{3*0.31}{12*6.38} = 0.01215, q = \frac{p*f_{yp}}{f_c} = \frac{0.01215*60.8}{5.5} = 0.1343$$
)  
P= 8m  $(\frac{1}{1-\frac{r}{L}} - 3 + 2\sqrt{2}) = 8*29.0*(\frac{1}{1-\frac{6}{72}} - 3 + 2\sqrt{2}) = 210 \text{ k}$ 

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